ALLAMA IQBAL OPEN UNIVERSITY, ISLAMABAD
(Department of Mathematics & Statistics)

WARNING

1. **PLAGIARISM OR HIRING OF GHOST WRITER(S) FOR SOLVING THE ASSIGNMENT(S) WILL DEBAR THE STUDENT FROM AWARD OF DEGREE/CERTIFICATE, IF FOUND AT ANY STAGE.**
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Course: Discrete Mathematics (3406)

Level: BS (CS) **Semester: Autumn, 2013**
Total Marks: 100 **Pass Marks: 40**

ASSIGNMENT No. 1
(Units 1–4)

Note: Attempt all questions, all questions carry equal marks

Q.1 a) Use symbols to write the logical form of the following arguments then use a truth table to test the arguments for validity; (05)
If Tom is not on team A, then Hua is on team B.
If Hua is not on team B, then Tom is on team A.
$\therefore$ Tom is not on team A or Hua is not on team B.

b) Construct circuit for the following Boolean expression (05)

$$(P \wedge Q) \vee \sim R$$

c) Determine whether the statement "the product of any two even integers is a multiple of 4." Is true or false? If it is true then Prove it else give a counter example. (05)

d) Show that $\sqrt{2}$ is an irrational number. (05)

Q.2 a) Write a negation for the following statements; (05)
i) $\forall$ Sets S, $\exists$ a set T such that $S \cap T = \varnothing$. Which is true the statement or its negation? Explain.
ii) $\exists$ a Set S such that $\forall$ sets T, $S \cup T = \varnothing$. Which is true the statement or its negative? Explain

b) Prove that for all sets A and B: (05)

$$A - (A \cap B) = A - B$$

c) A computer programming team has 14 numbers. (05)

i) How many ways can a group of 7 are chosen to work on a project?

ii) Suppose 8 team members are women and 6 are men; how many groups of 7 can be chosen that contain 4 women and 3 men?

d) Suppose that there are three roads from city A to city B and five roads from city B to city C; (05)

i) How many ways is it possible to travel from city A to city C via city B?

ii) How many different round trip routes are there from city A to B to C to B and back to A in which no road is traversed twice?

Q.3 a) Prove modus tollens. In other words, prove that the following argument form is invalid: (05)

$$p \to q$$
$$\sim p$$
$$\therefore \sim q$$

b) Write negation of the statement: (05)

$\forall$ animals x; if x is a cat then x has whiskers and x has claws.

c) Use modus and tollens to fill in valid conclusion for the following argument; (05)

All healthy people eat an apple a day.

Harry does not eat an apple a day.

$\therefore$ ____________________________ .

d) Indicate whether the following arguments are valid or invalid. Support your answer by drawing diagrams. (05)

All people are mice.

All mice are mortal.

$\therefore$ All people are mortal.

Q.4 a) What can you conclude about the validity OR invalidity of the following argument form? (10)

$$\forall x,\ \textit{if } P(x) \textit{ then } Q(x);$$
$$\sim P(a) \text{ for a particular } a$$
$$\therefore \sim Q(a)$$

b) Prove that for all positive integers a and b, $a|b$ if, and only if, $\gcd(a, b) = a$. (10)

Q.5 a) Write an algorithm to determine whether a given element x belongs to a given set, which is represented as an array $a[1], a[2], a[3], \ldots, a[n]$. (05)

b) For all integers a, b and c, if $a \mid b$ and $a \mid c$ *then* $a \mid (b + c)$. (05)

c) Use the well-ordering principal to prove that if a and b are any integers not both zero, then there exist integers u and v such that $\gcd(u, v) = ua + vb$. (05)

d) Write a negation for the following statements; (05)

i) $\forall$ Sets S, $\exists$ a set T such that $S \cap T = \varnothing$. Which is the true statement or its negation? Explain.

ii) $\exists$ *a* Set S such that $\forall$ sets T, $S \cup T = \varnothing$. Which is the true statement or its negation? Explain.

ASSIGNMENT No. 2

(Units 5–9)

Note: Attempt all questions, all questions carry equal marks

Q.1 a) A small town has only 500 residents. Must there be 2 residents who have the same birthday? Why? Explain your answer. (10)

b) Draw arrow diagrams for the Boolean functions defined by the following input/output table; (10)

Input		**Output**
P	**Q**	**R**
1	1	0
1	0	1
0	1	0
0	0	1

Q.2 a) Let $a_0, a_1, a_2, \ldots$ be the sequence defined recursively as follows $\forall$ K εZ (10)

$$a_K = a_{K-1} + 2 \qquad \text{if } K \geq 1$$
$$a_K = 1 \qquad \text{if } K = 0$$

Use Iteration to guess an explicit formula for the sequence.

b) A runner targets herself to improve her time on a certain course by *3* seconds a day. If on day O she runs the course in *3* minutes, how fast must she run it on the 14^{th} day to stay on target? (10)

Q.3 a) Refer to the following algorithm segment. For each positive integer n, let b_n be the number of iterations of the while loop; (10)

While ($n > 0$)
$n := n \text{ div } 3$
end while

Trace the action of this algorithm segment on n when the initial value of n is 424.

b) Prove that if a walk in a graph contains a repeated edge, then the walk contains a repeated vertex. (10)

Q.4 a) Suppose that there are three roads from city A to city B and five roads from city B to city C; (10)

i) How many ways is it possible to travel from city A to city C via city B?

ii) How many different round trip routes are there from city A to B to C to B and back to A in which no road is traversed twice?

b) A computer programming team has 14 members. (10)

i) Suppose two team members refuse to work together on projects. How many groups of 7 can be chosen to work on a project?

ii) How many ways can a group of 7 are chosen to work on a project?

iii) Suppose 8 team members are women and 6 are men; how many groups of 7 can be chosen that contain 4 women and 3 men?

Q.5 a) Draw all non-isomorphic graphs with four vertices and no more than two edges. (10)

b) Show that for any real number x, if $x > 1$ then $|2x^2 + 15x + 4| \leq 21|x^2|$ and use O- notation to express the result?